ON CORRECT INFERENCES
–BEYOND MATHEMATICAL LOGIC–

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Abstract. There are some possible definitions of correct inferences. In this paper, we shall introduce an axiom of correct inferences explicitly and prove some theorems which support that this axiom defines correct inferences uniquely in some restricted cases.

Logic is a study of correct inferences. Therefore, the main theme of logic is to answer the question: "What are correct inferences?" (Cf.[4] for general theories of inferences.)

In this paper, we shall try to answer this question by giving an axiom of correct inferences explicitly and prove theorems which support that this axiom defines correct inferences uniquely in some restricted cases.

Moreover, we shall try to explain why almost all persons are able to use logic in their ordinary lives without knowledge of Mathematical Logic.

In §1 below, we shall introduce some necessary concepts including two key concepts "front laws of inferences" and "back laws of inferences". By using these two key concepts, we shall introduce an axiom of correct inferences in §2.

In §1 and §2, we shall deal with languages used in our ordinary lives, called here "ordinary languages". But, in order to make more precise considerations on correct inferences, we need to introduce a second-order formal language in §3. By using this formal language, we shall state and prove some theorems in §4, which are useful for definitions of correct inferences.

In §5, we shall use ordinary languages again and prove some theorems. By using these theorems, we shall give two definitions of correct inferences. One is used in Mathematical logic and the other is used in our ordinary lives. Here, we shall investigate these two definitions of correct inferences and propose to use the definition used in our ordinary lives, generally.

§1.Preliminaries.

In this section and next section, we shall consider languages used in our ordinary lives. Such languages are called here "ordinary languages".

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Let L be an ordinary language which is used for communication. Let Inf(L) be the set of inferences in L. Then, we would like to define the set of correct inferences in L. But, inferences are used for communication in each society S which consists of individual persons. Hence, we shall deal with the set of inferences in Inf(L), which are "accepted by everybody in S". Such inferences are called here "S-correct inferences".

Here, we shall try to define the set of S-correct inferences by giving its axiom, for each S. Note that to define the set of S-correct inferences, it is necessary to make clear the meaning of the vague expression "accepted by everybody in S".

Let S^{∞} be the society which consists of all persons in past, present and future. Then, S^{∞}-correct inferences are called "logically correct inferences".

Mathematical Logic studied logically correct inferences and succeeded in defining and characterizing the set of logically correct inferences in Inf(L), in case that L is a first-order language by Godel Completeness Theorem.

Premises and the conclusion of an inference in L are sentences which express information. In order to know information expressed by a sentence F, it is necessary to know the truth value of F. On the other hand, in order to know the truth value of F, it is necessary to know meanings of all words in F.

Now, we consider the sentence "Socrates is honest". Then, there are three words "Socrates", "is" and "honest" in this sentence. Here, "is" is a word to denote the subject-predicate relation in sentences and its meaning is common for everybody.

Words whose meanings are common for everybody, are called "logical words". Then, "is" an example of logical words. Also, "¬"(not), "∧"(and), "∨"(or) , "∀"(all, any, every) and "∃"(exist) are examples of logical words.

On the other hand, two words "Socrates" and "honest" are words whose meanings depend on persons who use them. Such words are called "non-logical words".

Usually, the word "Socrates" is used to denote an object c in a set U called here "universe" and word "honest" is used to denote a property P on the same universe U.

In this situation, c is a meaning of the word "Socrates" and P is a meaning of the word "honest". Furthermore, c is an element of U and P is a property on U. Therefore, meanings of "Socrates" and "honest" are given by designating a non-empty set U, an element c of U and a property P on U.

Hence, U with c and P is called "meaning structure (m-structure)" of this sentence and denoted by <U;c,P>. In case that c has the property P, we say that "<U;c,P> satisfies this sentence".

Similarly, truth values of sentences are decided by their m-structures. On the other

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hand, m-structures of sentences are chosen by persons who use them. In case that a person p chooses an m-structure which satisfies F, we say that "F is true for p" or "p admits F to be true".

Sometimes, we identify each society S with the set of m-structures chosen by some members in S.

Sentences which are true for all persons in a society S, are called "S-true sentences". Also, S^{∞}-true sentences are called "logically true sentences".

If the conclusion of an inference is obtained by applying a general law to premises of the inference, then the general law is called "back law" of the inference.

For example, the general law "Every Greek is honest" is a back law of the inference from "Socrates is Greek" to "Socrates is honest", because the sentence "Socrates is honest" is obtained by applying the general law "Every Greek is honest" to the sentence "Socrates is Greek".

Back laws of the inference from "Socrates is Greek" to "Socrates is honest" are the following seven laws: ∀x(¬(x is Greek)∨(x is honest)) (= "Every Greek is honest"),
∀X(¬(Socrates is X)∨(Socrates is honest)), ∀Y(¬(Socrates is Greek)∨(Socrates is Y)),
∀X∀x(¬(x is X)∨(x is honest)), ∀Y∀x(¬(x is Greek)∨(x is Y)),
∀Y∀X(¬(Socrates is X)∨(Socrates is Y)), ∀Y∀X∀x(¬(x is X)∨(x is Y)),
where x is an individual variable and X,Y are predicate variables.

An exact definition of "back law" will be given in §3 below.

Although there are no explicit definitions of correct inferences, we can assume the following (*), because any inference which has an S-true back law should be accepted by everybody in S:

(*) Any inference which has an S-true back law is S-correct.

Let d be an inference in L and let S be a society. Then, the sentence

"Anybody in S who admits all premises of d to be true,
has to admit the conclusion of d to be true"

is called "S-front law" of d.

Suppose that there is an S-correct inference d whose S-front law is not true. Then、 d is accepted by everybody in S, but there is a person in S who admits all premises of d to be true but does not admit the conclusion of d to be true, This is impossible.

This means that we can assume the following (**):

(**) Any S-correct inference has the true S-front law.

Note that the S-front law of d is a sentence in a meta-language of L. Therefore, the truth value of the S-front law of d is independent of persons in S.

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§2.Axiom of S-correct inferences.

Let d be an inference in L and let S be a society. We use d as a variable and S as a constant or parameter in this section. Therefore, “d has an S-true back law”, “d is S-correct” and “d has the true S-front law” are conditions on d, which depend on S.

By (*) and (**), we can introduce the following axiom of S-correct inferences.

Axiom of S-correct inferences: *Any inference which has an S-true back law is S-correct and any S-correct inference has the true S-front law.*

This axiom means that the unknown condition “d is S-correct” is a necessary condition of the known condition “d has an S-true back law” and sufficient condition of the known condition “d has the true S-front law”.

Therefore, this axiom is a condition on condition, whose variable is the unknown condition “d is S-correct”. Hence, this axiom is a meta-condition. A condition (on d) which satisfies this meta-condition is called “candidate” for S-correct inferences. Then, two conditions “d has an S-true back law” and “d has the true S-front law” are candidates. Moreover, “d has an S-true back law” is a sufficient condition of every candidate and “d has the true S-front law” is a necessary condition of every candidate.

Therefore, we would like to investigate relations between “d has an S-true back law” and “d has the true S-front law”.

Now, consider the following principle called here “Back Law Principle (BLP)” which is a condition on S.

BLP *: Any inference whose S-front law is true, has an S-true back law.*

Suppose that S satisfies BLP. Then, there is only one candidate, which is “d has an S-true back law” or “d has the true S-front law”, because three conditions “d has an S-true back law”, “d is S-correct” and “d has the true S-front law” are equivalent as conditions on d.

Hence, we would like to find some conditions on S which satisfy BLP.

To find these conditions, we shall consider the inference from “Socrates is Greek” to “Socrates is honest”.

Let S be a society and let <U(p); c(p),P(p),Q(p)> be the m-structure chosen by p, for each p in S. Here, <U(p); c(p),P(p),Q(p)> is an m-structure of two sentences “Socrates is Greek” and “Socrates is honest”. Therefore, if c(p) has the property P(p), then p admits the sentence “Socrates is Greek” to be true, and if c(p) has the property Q(p), then p admits the sentence “Socrates is honest” to be true.

Moreover, we assume that all persons in S have the same universe U, the same meaning P of the word “Greek” and the same meaning Q of the word “honest”.

Therefore, U(p)=U, P(p)=P and Q(p)=Q for any p in S.

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In the following, P and Q are identified with sets {x ε U; x has the property P} and {x ε U; x has the property Q}, respectively.

CASE 1: We assume that there are three elements c, d, e in U and two non-empty subsets S_1 ,S_2 of S, such that :

①; c ε $P \cap Q^c$, ②; d ε $P \cap Q$ and c(p)=d for any p in S_1.

③; e ε $P^c \cap Q^c$ and c(p)=e for any p in S_2.

④; $S_1 \cup S_2$ = the empty set and $S_1 \cup S_2$=S.

By ②,③,④, the S-front law is true.

By ①, the back law ∀x(¬(x is Greek)∨(x is honest)) is not S-true.

By ③, the back law ∀X(¬(Socrates is X)∨(Socrates is honest)) is not S-true.

By ②, the back law ∀Y(¬(Socrates is Greek)∨(Socrates is Y)) is not S-true.

Hence, other four back laws are also not S-true. Therefore, we can conclude that the S-front laws of this inference is true, but there are no S-true back laws of this inference.

CASE 2: We assume that

⑤; for any c in U, there is a person p in S such that c=c(p).

In this case, we can show that if the S-front law of this inference is true, then it has an S-true back law.

(Proof) Assume that the S-front law of the inference from "Socrates is Greek" to "Socrates is honest" is true. We would like to show that the back law "Every Greek is honest" is S-true.

Let u be an arbitrary element in U such that u ε P. Then there is a person p in S such that u=c(p) by ⑤. Since the S-front law of this inference is true, we have u ε Q.

Hence, the back law "Every Greek is honest" is S-true.

(Q.E.D.)

In case 1, the inference "Socrates is Greek" to "Socrates is honest" has its true S-front law but does not have any S-true back law.

In case 2, if this inference has its true S-front law, then it always has an S-true back law.

In case 1, the set {c(p); p ε S } is a proper subset of U. On the other hand, in case 2, the set {c(p); p ε S } is equal to U.

If {c(p); p ε S }=U, then we say that the non-logical word "Socrates" has diversity in S. (An exact definition of diversity will be given in §4 below.)

In case 2, "Socrates" has diversity in S, but in case 1, does not have.

Let S be a society such that every non-logical word in L has diversity in S.

Then, we can expect that S satisfies BLP. We would like to describe these

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phenomena. But, ordinary languages are not convenient for this purpose. So, in the following two sections, we shall use a second-order formal language instead of ordinary languages.

§3.Formal language.

Suppose that L is a second-order formal language whose non-logical constant symbols are individual constant symbols and predicate constant symbols.

Also, variable symbols used in L are individual variable symbols and predicate variable symbols. We assume that readers are familiar with formal languages. (Cf. Chang and Keisler [1].)

But, here, we would like to remind of the satisfaction relation between structures for L (L-structures) and sentences (closed formulas) in L.

An L-structure M consists of the followings:

a non-empty set called "universe" of M and denoted by $|M|$,

an element M(c) in $|M|$, for each individual constant symbol c,

a subset M(P) of $|M|^m$, for each m-ary predicate constant symbol P.

So, an L-structure M is denoted by $<|M|;M(c),...,M(P),...>$.

Let F be a sentence in L and let M be an L-structure. Then, the meta-sentence F[M] in a meta-language of L, is obtained by interpreting as follows:

Individual variable symbols are interpreted as individual variables on $|M|$,

Predicate variable symbols are interpreted as predicate variables on $|M|$,

Individual constant symbols c,… are interpreted as individual constants M(c),...,

Predicate constant symbols P,... are interpreted as predicate constants M(P),....

If the sentence F[M] is true, we say that "M satisfies F" or "M is a model of F".

The sentence F[M] expresses a relation between M and F. This relation is called "satisfaction relation".

Let W be a subclass of the class St(L) and let F be a sentence in L, where St(L) is the class of L-structures.

Then, a sentence F in L is said to be "W-true" if every L-structure in W satisfies F.

DEFINITION 1 (DEDUCIBILITY). *A sentence F in L is deducible from sentences $F_1,F_2,...,F_n$ in L if any L-structure M which satisfies all $F_1,F_2,...,F_n$, always satisfies F.*

DEFINITION 2 (W-FRONT LAW). *Let d be an inference from $F_1,F_2,...,F_n$ to F. Then, the W-front law of d is the following sentence in a meta-language of L:*

"Any L-structure M in W which satisfies all $F_1,F_2,...,F_n$, always satisfies F"

DEFINITION 3 (KERNEL). *For each inference d from $F_1,F_2,...,F_n$ to F, the sentence*

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$\neg F_1 \vee \neg F_2 \vee \ldots \vee \neg F_n \vee F$ is called "kernel" of d and denoted by Ker(d).

Note that Ker(d) is a sentence in L and the W-front law of d is a sentence in a meta-language of L.

Then, we have the following two facts easily.

FACT 1. *A sentence F is deducible from sentences $F_1,F_2,\ldots,F_n$ in L if and only if the St(L)-front law of the inference from $F_1,F_2,\ldots,F_n$ to F, is true.*

FACT 2 (DEDUCTION THEOREM). *The W-front law of d is true if and only if Ker(d) is W-true, for any inference d and any subclass W of St(L).*

Next, we introduce some necessary notions.

DEFINITION 4 (CLOSURES, LOGICAL CLOSURES). *Suppose that $F(\alpha_1,\alpha_2,\ldots,\alpha_n)$ is a sentence in L and $\alpha_1,\alpha_2,\ldots,\alpha_n$ ($n\geq 1$) are distinct non-logical constant symbols which appear in F. Then, the sentence $\forall\xi_1 \forall\xi_2\ldots \forall\xi_n F(\xi_1,\xi_2,\ldots,\xi_n)$ is called "closure" of F. A closure of F which has no non-logical constant symbol is called "logical closure" of F.*

For example, closures of the sentence "$\neg P(c) \vee Q(c)$" are the following seven sentences: "$\forall x(\neg P(x)\vee Q(x))$", "$\forall X(\neg X(c)\vee Q(c))$", "$\forall Y(\neg P(c)\vee Y(c))$", "$\forall X\forall x(\neg X(x)\vee Q(x))$", "$\forall Y\forall x(\neg P(x)\vee Y(x))$", "$\forall X\forall Y(\neg X(c)\vee Y(c))$", "$\forall X\forall Y\forall x(\neg X(x)\vee Y(x))$" and its logical closure is "$\forall X\forall Y\forall x(\neg X(x)\vee Y(x))$".

Note that any α_i does not occur in $\forall\xi_1\forall\xi_2\ldots\forall\xi_n F(\xi_1,\xi_2,\ldots,\xi_n)$, for each i=1,...,n. Therefore, "$\forall x(\neg P(x)\vee Q(c))$", "$\forall x(\neg P(c)\vee Q(x))$" are not closures of "$\neg P(c)\vee Q(c)$".

Then, we have the following facts:

FACT 3. *A closure of a closure of F is a closure of F.*

FACT 4. *F is deducible form any closure of F.*

Here, we would like to introduce "General Law Sentences" and "Back Laws of Inferences".

DEFINITION 5 (GENERAL LAW SENTENCES). *Sentences of the form*

$$\forall\xi_1 \forall\xi_2\ldots \forall\xi_n(\neg F_1(\xi_1,\xi_2,\ldots,\xi_n)\vee\neg F_2(\xi_1,\xi_2,\ldots,\xi_n)\vee\ldots\vee\neg F_m(\xi_1,\xi_2,\ldots,\xi_n)\vee F(\xi_1,\xi_2,\ldots,\xi_n))$$

are called "General Law Sentences", where $n\geq 1$ and $F_1,F_2,\ldots,F_m,$ F are formulas in L.

Every general law sentence expresses a general law in each L-structure.

For example, a general law sentence:

$$\forall\xi_1\forall\xi_2\ldots\forall\xi_n(\neg F_1(\xi_1,\xi_2,\ldots,\xi_n)\vee\neg F_2(\xi_1,\xi_2,\ldots,\xi_n)\vee\ldots\vee\neg F_m(\xi_1,\xi_2,\ldots,\xi_n)\vee F(\xi_1,\xi_2,\ldots,\xi_n))$$

expresses the following general law in each L-structure M.

"Any $\xi_1,\xi_2\ldots,\xi_n$ which satisfy all

$F_1(\xi_1,\xi_2,\ldots,\xi_n),F_2(\xi_1,\xi_2,\ldots,\xi_n),\ldots$and $F_m(\xi_1,\xi_2,\ldots,\xi_n)$, always satisfy $F(\xi_1,\xi_2,\ldots,\xi_n)$"

DEFINITION 6 (BACK LAWS OF INFERENCES). *Let d be an inference in L. Then, a general law sentence:*

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$\forall\xi_1 \forall\xi_2 \ldots \forall\xi_n(\neg F_1(\xi_1,\xi_2,\ldots,\xi_n)\vee\neg F_2(\xi_1,\xi_2,\ldots,\xi_n)\vee\ldots\vee\neg F_m(\xi_1,\xi_2,\ldots,\xi_n)\vee F(\xi_1,\xi_2,\ldots,\xi_n))$

is called "Back Law" of d if there are distinct non-logical constant symbols $a_1,a_2,\ldots,a_n$, *which do not appear in this sentence such that the premises of d are* $F_1(a_1,a_2,\ldots,a_n),F_2(a_1,a_2,\ldots,a_n),\ldots,F_m(a_1,a_2,\ldots,a_n)$ *and the conclusion of d is* $F(a_1,a_2,\ldots,a_n)$.

For example, "∀x(¬P(x)∨Q(x))" is a back law of the inference from "P(c)" to "Q(c)", but "∀x(¬P(x)∨Q(c))" and "∀x(¬P(c)∨Q(x))" are not back laws of this inference,

In this case, we can obtain "Q(c)" by applying the general law "∀x(¬P(x)∨Q(x))" to "P(c)". In general, if we can obtain the conclusion of an inference d by applying a general law F to the premises of d, then we says that F is a back law of d.

Definition 6 is a precise expression of this fact. (cf.[2])

Then, we have the following theorem:

FACT 5 (REPRESENTATION THEOREM OF BACK LAWS). *Let d be an inference in L. Then, every back law of d is a closure of Ker(d) and every closure of Ker(d) is a back law of d.*

This theorem shows that "back laws of d" and "closures of Ker(d)" are the same.

By Fact 2, Fact 4 and Fact 5, we have:

FACT 6. *If d has a W-true back law, then the W-front law of d is true, for any inference d in L and any subclass W of St(L).*

§4.Main Theorems.

Let d be an inference in L and let W be a subclass of St(L). We use d as a variable and W as a constant or parameter, in this section.

Here, we introduce a key notion of this paper.

DEFINITION 7 (DIVERSITY OF NON-LOGICAL CONSTANTS).

(i) Let "c" be an individual constant symbol in L and let W be a subset of St(L). Then, "c" has Diversity in W if for any L-structure M in W and any element u in the universe $|M|$ *of M, there is an L-structure* $M^\#$ *in W such that* $|M^\#|=|M|$, $M^\#(c)=u$ *and* $M^\#(\alpha)=M(\alpha)$ *for each non-logical constant symbol α except "c".*

(ii) Let "P" be an m-ary predicate constant symbol in L and let W be a subset of St(L). Then, "P" has Diversity in W if for any L-structure M in W and any subset V of $|M|^m$, *there is an L-structure* $M^\#$ *in W such that* $|M^\#|=|M|$, $M^\#(P)=V$ *and* $M^\#(\alpha)=M(\alpha)$ *for each non-logical constant symbol α except "P".*

Let M be a fixed L-structure. Then, an individual constant symbol "c" has Diversity in the set $\{M^\#;|M^\#|=|M|$ and $M^\#(\alpha)=M(\alpha)$ for each non-logical constant symbol α except "c"$\}$. Also, a predicate constant symbol "P" has Diversity in the set $\{M^\#;|M^\#|=|M|$ and $M^\#(\alpha)=M(\alpha)$ for each non-logical constant symbol α except "P"$\}$.

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Moreover, every non-logical constant symbol has diversity in $\{M^{\#}; |M^{\#}|=|M|\}$.

Of course, every non-logical constant symbol has diversity in St(L).

Then, we have the following theorem.

THEOREM 1 (DIVERSITY THEOREM). *Let "α" be a non-logical constant symbol in L and F(α) is a sentence in L. Assume that W is a subclass of St(L) and "α" has Diversity in W. Then,*

(i) F(α) is true for some M in W if and only if $\exists\xi F(\xi)$ *is true for some M in W.*

(ii) F(α) is true for any M in W if and only if $\forall\xi F(\xi)$ *is true for any M in W.*

(Proof) At first, we assume that "α" is an individual constant symbol "c".

Assume that "c" has Diversity in W. Then, "only if part" of this theorem is obvious because $\exists xF(x)$ is deducible from F(c).

So, it is sufficient to show the "if part" of this theorem.

Assume that $\exists xF(x)$ is true for some M in W. Then, there is u in the universe of M such that F(c) is true for $M^{\#}$, where $|M^{\#}|=|M|$, $M^{\#}(c)=u$ and $M^{\#}(\alpha)=M(\alpha)$ for each non-logical constant symbol α except "c". Since "c" has Diversity in W, this $M^{\#}$ belongs to W. This shows that F(c) is true for some L-structure in W.

We can prove the case that α is a predicate constant symbol, similarly. So we omit it.

We can prove (ii) by taking negations of both side of (i).

(Q.E.D.)

By this theorem, we can easily prove the following theorem.

THEOREM 2 (ELIMINATION THEOREM OF UNIVERSAL QUANTIFIERS). *Assume that* $\forall\xi_1\forall\xi_2...\forall\xi_n F(\xi_1,\xi_2,...,\xi_n)$ *is a closure of the sentence* $F(\alpha_1,\alpha_2,...,\alpha_n)$ *in L and all* $\alpha_1,\alpha_2,...,\alpha_n$ *have Diversity in W. Then,*

$\forall\xi_1\forall\xi_2...\forall\xi_n F(\xi_1,\xi_2,...,\xi_n)$ *is W-true if and only if* $F(\alpha_1,\alpha_2,...,\alpha_n)$ *is W-true.*

Note that Theorem 2 above is well-known in case that W=St(L).

THEOREM 3. *Let d be an inference and let* $\alpha_1,\alpha_2,...,\alpha_n$ $(n\geq 1)$ *be distinct non-logical constant symbols which appear in d. Also let* $F(\alpha_1,\alpha_2,...,\alpha_n)$ *be Ker(d).*

If the W-front law of d is true and $\alpha_1,\alpha_2,...,\alpha_n$ *have Diversity in W, then the sentence* $\forall\xi_1\forall\xi_2...\forall\xi_n F(\xi_1,\xi_2,...,\xi_n)$ *is a W-true back law of d.*

(Proof) Suppose that the W-front law of d is true and $\alpha_1,\alpha_2,...,\alpha_n$ *have Diversity in W.*

Then, $F(\alpha_1,\alpha_2,...,\alpha_n)$ is W-true by Fact 2. Since $\alpha_1,\alpha_2,...,\alpha_n$ have diversity in W, $\forall\xi_1\forall\xi_2...\forall\xi_n F(\xi_1,\xi_2,...,\xi_n)$ is W-true by Theorem 2.

By Fact 5, $\forall\xi_1\forall\xi_2...\forall\xi_n F(\xi_1,\xi_2,...,\xi_n)$ is an W-true back law of d.

(Q.E.D.)

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§5.Conclusions.

Here, we come back to ordinary languages. In this section, we shall make some informal but practical investigations on correct inferences. We use some results in §3 and § 4, by interpreting ordinary languages into the second-order language L introduced in §3, naturally.

Also, we identify each m-structure with the corresponding L-structure. Then, each society S is identified with the class W of corresponding L-structures.

Note that the society S^{∞} is identified with St(L).

Then, Theorem 3 in §4 can be rewritten in the following:

THEOREM 3#. *Let d be an inference and let* $a_1, a_2, \ldots, a_n$ $(n \geq 1)$ *be distinct non-logical words which appear in d. Also let* $F(a_1, a_2, \ldots, a_n)$ *be Ker(d).*

If the S-front law of d is true and $a_1, a_2, \ldots, a_n$ *have Diversity in S, then the sentence* $\forall\xi_1 \forall\xi_2 \ldots \forall\xi_n F(\xi_1, \xi_2, \ldots, \xi_n)$ *is an S-true back law of d.*

Now, we would like to consider a new principle which is obtained from BLP by restricting inferences there to those which are useful for communication.

In order to express this principle, we need some new notions.

Words whose meaning are common for everybody in S, are called "S-logical words" and words which are not S-logical words, are called "S-non-logical words".

Of course, all logical words are S-logical words and all S-non-logical words are non-logical words.

Sentences in L which consist of S-logical words only, are called "S-logical sentences".

For example, if "Greek" and "honest" are S-logical words, then the sentence "Every Greek is honest" is an S-logical sentence.

Here, we consider an inference d which consists of S-logical sentences only.

Then, this inference d is useless for communication in S, because truth values of all premises of d and the conclusion of d are common for everybody in S. In fact, S-correct inferences are used in cases that there are some persons in S and some sentences such that they do not know truth values of these sentences but want to know.

For example, assume that a person p in S wants to know whether a sentence F is true or not. If p can find an S-correct inference d whose conclusion is F and all premises of d are true for p, then p can confirm that F is true.

In fact, if "Greek", "honest" and "Socrates" are S-logical words, then the inference from "Socrates is Greek" to "Socrates is honest" is useless for communication.

Inferences which consist of S-logical sentences only, called "S-trivial". Inferences which are not S-trivial, are called "S-non-trivial". Moreover, inferences which consist

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of logical words only, are called "trivial". Inferences which are not trivial, are called "non-trivial".

Assume that "Greek" and "honest" are S-logical words. If the word "Socrates" is S-logical, then the inference from "Socrates is Greek" to "Socrates is honest" is S-trivial. But, if the word "Socrates" is S-non-logical, then this inference is S-non-trivial.

Then, S-trivial inferences are useless for communication in S and S-non-trivial inferences are useful for communication in S.

Now, we consider the following BLP$^{\#}$ which is obtained from BLP by restricting inferences to S-non-trivial inferences.

BLP$^{\#}$. Any S-non-trivial inference whose S-front law is true, has an S-logical and S-true back law.

It is very difficult to obtain a condition (on S) which satisfies BLP, but it is possible to find a condition (on S) which satisfies BLP$^{\#}$ above.

Let d be an S-non-trivial inference whose S-front law is true.

Assume that $\alpha_1,\alpha_2,\ldots,\alpha_n$ $(n\geq 1)$ is a list of all distinct S-non-logical words which appear in d. Also let $F(\alpha_1,\alpha_2,\ldots,\alpha_n)$ be Ker(d). Then $\forall\xi_1\forall\xi_2\ldots\forall\xi_n F(\xi_1,\xi_2,\ldots,\xi_n)$ is an S-logical back law of d.

Assume that $\alpha_1,\alpha_2,\ldots,\alpha_n$ have Diversity in S. Then, by Theorem 3$^{\#}$, the sentence $\forall\xi_1\forall\xi_2\ldots\forall\xi_n F(\xi_1,\xi_2,\ldots,\xi_n)$ is an S-logical and S-true back law of d. Hence, we have the following theorem:

THEOREM 4. *Assume that every S-non-logical word has Diversity in S. Then, S satisfies BLP$^{\#}$.*

On the other hand, by rewriting Fact 6, we have:

FACT 6$^{\#}$. *If d has an S-true back law, then the S- front law of d is true.*

From Theorem 4 and Fact 6$^{\#}$, we have the following theorem.

THEOREM 5. *Assume that every S-non-logical word has Diversity in S. Let d be an S-non-trivial inference. Then, the S-front law of d is true if and only if d has an S-true back law.*

Since every non-logical word has diversity in S^{∞}, we have:

THEOREM 6. *Let d be a non-trivial inference. Then, the S^{∞}-front law of d is true if and only if d has a logically true back law.*

Here by using two theorems above, we are able to consider definitions of correct inferences. There are two possible definitions of correct inferences which satisfy the axiom in §3.

Definition A. *An inference is S-correct if its S-front law is true.*

Definition B. *An inference is S-correct if it has an S-true back law.*

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Definition A uses “front laws” and Definition B uses “back laws”, to define correct inferences. We have shown that these two definitions are equivalent in cases that inferences and societies are restricted. (Cf. Theorem 5.)

Mathematical Logic uses “front laws” and Definition A. (Cf. Fact 1.) But, in this case these two definitions are equivalent if we neglect trivial inferences. (Cf. Theorem 6.) This means that we can use Definition B instead of Definition A, in Mathematical Logic.

On the other hand, almost all persons use “back laws” and Definition B, unconsciously in their ordinary lives. In fact, laws and rules used in our ordinary lives are examples of back rules of inferences used there.

This is the main reason why almost all persons are able to use logic without knowledge of Mathematical Logic.

Also, we have the belief that correctness of inferences should be supported by general laws accepted by everybody. (Cf. [2], [3]).

Therefore, we would like to propose to use Definition B, generally.

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